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Non-Local Modification of Gravity and the Cosmological Constant Problem

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Abstract

We propose a phenomenological approach to the cosmological constant problem based on generally covariant non-local and acausal modifications of four-dimensional gravity at enormous distances. The effective Newton constant becomes very small at large length scales, so that sources with immense wavelengths and periods — such as the vacuum energy— produce miniscule curvature. Conventional astrophysics, cosmology and standard inflationary scenaria are unaffected, as they involve shorter length scales. A new possibility emerges that inflation may "self-terminate" naturally by its own action of stretching wavelengths to enormous sizes. In a simple limit our proposal leads to a modification of Einstein's equation by a single additional term proportional to the average space-time curvature of the Universe. It may also have a qualitative connection with the dS/CFT conjecture.

1 Introduction

The Cosmological Constant Problem (CCP) is one of the most pressing conceptual problems in physics. The energy-momentum tensor $T_{\mu\nu}$ is expected to contain a vacuum energy density piece $\mathcal{E} g_{\mu\nu}$, and the natural value for $\mathcal{E}$ coming from the Standard Model sector should be at least $\sim (\mathrm{TeV})^4$. However, according to the Einstein equations

$$M_{\mathrm{Pl}}^2 G_{\mu\nu} = T_{\mu\nu} \,, \tag{1}$$

such a vacuum energy would give rise to a drastically different cosmology than what we observe. For instance, if $\mathcal{E} > 0$, the universe quickly becomes asymptotically de Sitter with a radius of curvature $\sim$ mm, while the observed curvature radius of the Universe is enormously larger, $\sim H_0^{-1} \sim 10^{28}$ cm.

The most familiar formulation of the CCP is "Why is the vacuum energy so small?". This reflects the most common approach to the problem: to invoke some dynamics, analogous to the Peccei-Quinn mechanism for the strong CP problem, that is flexible enough to adjust and cancel any value of the vacuum energy (see for a review Ref. [1]). In this formulation, the mystery is even further deepened by recent cosmological observations [2] suggesting that the Universe has recently entered an accelerated phase with the curvature radius $\sim H_0^{-1}$, which is usually ascribed to a tiny $\mathcal{E} \sim (\mathrm{mm})^{-4}$. The question then becomes "why is the vacuum energy density so tiny, 60 orders of magnitude smaller than its natural value, but not zero"?

However, a more precise formulation of the problem is: "Why does the vacuum energy gravitate so little?". This suggests an approach where $\mathcal{E}$ keeps its natural value $\sim (\mathrm{TeV})^4$, but gravity is modified so that this large vacuum energy density does not give rise to large observable curvature (for attempts see [3, 4]).

More specifically, it is natural to try and modify gravity in the *infrared* (IR) to address the CCP, given that the CCP seems to be associated with very low energy scales with respect to the Planck scale or even the weak scale. However, until recently there were no explicit examples of consistent theories where the behaviour of gravity is classically modified at large distances. This situation changed with the advent of models where the Standard Model fields are localised to a brane in infinite volume extra dimensions [5], where gravity on the brane transitions from being four-dimensional to higher-dimensional at very large distances. These models motivated afresh the possibility of addressing the cosmological constant problem by IR modification of gravity [6], where the vacuum energy (brane tension) mostly curves the bulk, while ordinary gravity is trapped to the brane at observable distances by the presence of a large Einstein-Hilbert action localised on the brane. A specific proposal along these lines was made in Ref. [7], where it is argued that the graviton propagator is modified in the infrared in such a way that large wavelength sources, such as the vacuum energy, gravitate very weakly. As a result, even huge vacuum energy does not curve space. On the other hand, short wavelength sources, such as planets, stars, galaxies and clusters gravitate (almost) normally.

It is challenging to perform computations in the framework of [7] in order to explore its consequences in a realistic setting. However the general idea of addressing the CCP with non-local modification of gravity in the infrared is so attractive that it is desirable to try and explore the viability of this idea, as well as the properties such non-local modifications should have, in a concrete way.

This leads us to ask a more modest question: What should an effective, four-dimensional, long-distance, classical description of physics incorporating non-locality look like, in order to address the CCP in a realistic setting?

2 Newton's Constant as a High-Pass Filter

The fundamental physical idea we want to implement, inspired by the example of [7], is to make the effective Newton constant depend on the frequency and wavelength in such a way that for sources that are uniform in both space and time, such as the vacuum energy, the effective Newton constant is tiny, shutting off their gravitational effects. Analogs in electromagnetism are frequency-dependent dielectrics and high-pass filters.

We will not attempt to derive this physics from a consistent quantum effective field theory. Indeed, the modifications we will end up considering are non-local and acausal, and are hard to imagine coming from conventional field theories. Instead, we will simply modify Einstein gravity at the level of classical equations of motion, implementing the physical idea of "the Newton constant as a high-pass filter", with the goal of resolving the CCP without any fine adjustments of the parameters in the equations of motion, while retaining all the usual successes of general relativity. As we will see, this can be accomplished with extremely simple non-local modifications of the equations of motion. We will not attempt to derive our equations of motion from a variational principle; this is no great loss since any action principle itself would have to be non-local and would not necessarily directly lead to a sensible quantum theory in any case. Also the example of [7] suggests that the *local* action formulation may require going beyond the four dimensional theory.

A first example of a modified Einstein equation that incorporates the above-mentioned properties takes the form

$$M_{\rm Pl}^2 \left(1 + \mathcal{F}(L^2\,\nabla^2)\right) G_{\mu\nu} = T_{\mu\nu}\,, \tag{2}$$

where $\mathcal{F}(L^2\,\nabla^2)$ is the "filter function" with the following properties:

$$\mathcal{F}(\alpha) \to 0 \quad \text{for} \quad \alpha \gg 1\,; \tag{3}$$

$$\mathcal{F}(\alpha) \gg 1 \quad \text{for} \quad \alpha \ll 1\,. \tag{4}$$

L in (2) is a distance scale at which gravity is modified; it can be infinite, or very large but finite; $\nabla^2 \equiv \nabla_\mu \nabla^\mu$ denotes the covariant d'Alambertian. One can think of (2) as the Einstein equation with the effective Newton constant $(8\pi G_N^{\text{eff}})^{-1} = M_{\text{Pl}}^2(1+\mathcal{F})$. It is immediately clear that at least for the case where $T_{\mu\nu}$ is pure vacuum energy density $\mathcal{E} g_{\mu\nu}$, the maximally symmetric solution to these equations of motion can have acceptably small curvature if $\mathcal{F}(0)$ is large enough. This is because in a maximally symmetric space $G_{\mu\nu} = -g_{\mu\nu}R/4$, where R is the (space-time constant) Ricci scalar. Since $g_{\mu\nu}$ is covariantly conserved, $\nabla^2 g_{\mu\nu} = 0$, so

$$M_{\text{Pl}}^2 \left(1 + \mathcal{F}(L^2\nabla^2)\right) G_{\mu\nu} = \left(M_{\text{Pl}}^2 + \bar{M}^2\right) G_{\mu\nu} \,, \tag{5}$$

where

$$\bar{M}^2 = \mathcal{F}(0)\, M_{\text{Pl}}^2 \gg M_{\text{Pl}}^2 \,, \tag{6}$$

and therefore

$$R = -\frac{4\mathcal{E}}{M_{\text{Pl}}^2 + \bar{M}^2} \,, \tag{7}$$

which can be sufficiently small provided that $\bar{M}$ is sufficiently large. Note that this does not require any fine adjustments for $\mathcal{E}$ or $\bar{M}$ so long as $\bar{M}$ is large enough.

It is also instructive to see how this effective suppression of the cosmological constant is seen in a linearised approximation about flat space; usually, the $\mathcal{E}$ generates a tadpole for the graviton. The effect of the $\mathcal{F}(L^2\nabla^2)$ term is to modify the graviton propagator in momentum space to (neglecting indices) $(1+\mathcal{F}(k^2L^2))^{-1}1/k^2$; as $\mathcal{F}(0)$ is made large, this shuts off the propagator at zero external momentum and removes the effect of the tadpole completely in the $\mathcal{F}(0) \to \infty$ limit. This is a generalisation of the propagator of Ref. [7]. This is not surprising, since, as we said above, this theory modifies gravity in far infrared.

It is also interesting that this kind of modified propagator can arise without using branes or extra dimensions in an important way, but instead by perturbatively modifying the world-sheet action in string theory [8]. Such modifications are used to eliminate the effect of tadpoles generated in non-SUSY string theories, leading to new perturbative stringy backgrounds which are static and non-supersymmetric [8].

3 $L \to \infty$

3.1 A Concrete Example

After this motivation, we now examine the physical consequences of this idea in more detail. To do so, it is convenient to consider the simplest possible modification with the desired properties, which we can loosely motivate by taking the $L \to \infty$ limit in Eq. (2). Roughly speaking, the $\mathcal{F}(L^2\nabla^2)G_{\mu\nu}$ piece will then extract the space-time "zero mode" of $G_{\mu\nu}$; a zero mode $\psi_{\mu\nu}$ would satisfy $\nabla^2\psi_{\mu\nu} = 0$, which always has a generic solution $\psi_{\mu\nu} = g_{\mu\nu}$ since $g_{\mu\nu}$ is covariantly constant. (For our heuristic purposes here we assume that we are working with a Euclidean metric; for Minkowski space there are additional solutions corresponding to the excitation on the light-cone). So as L goes to infinity, we can replace $\mathcal{F}(L^2\nabla^2)G_{\mu\nu}$ by the "zero mode" part of $G_{\mu\nu}$, which is proportional to $g_{\mu\nu}$. The constant of proportionality can be determined by taking trace, and we arrive at the equation of motion

$$M_{\rm Pl}^2 \, G_{\mu\nu} \, - \, \frac{1}{4} \, \bar{M}^2 \, g_{\mu\nu} \bar{R} \, = \, T_{\mu\nu} \, , \tag{8}$$

where

$$\bar{R} \, \equiv \, \frac{\int \, d^4x \, \sqrt{g} \, R}{\int \, d^4x \, \sqrt{g}} \tag{9}$$

is the space-time averaged Ricci curvature and $\bar{M}$ is defined in (6).

The above "derivation" was mainly intended to motivate this equation of motion. It is interesting that Eq. (8) is universal, independent of the filter function $\mathcal{F}$. We could have heuristically argued for (8) as follows: in the limit $L \to \infty$ the filter excises only the infinite wavelength and period fluctuations from the dynamics. These involve the space-time Fourier transform at vanishing momentum and frequency, i.e., the space-time average of dynamical quantities. General covariance dictates that we only consider space-time averages of scalars. Furthermore, the simplest dynamical scalar in gravity is the curvature scalar. This suggests that we modify Einstein's equations by a term proportional to the space-time average of the curvature scalar. But this is precisely what Eq. (8) does. The electromagnetic analogue of Eq. (8) is:

$$\nabla \cdot \vec{E} \, + \, \bar{\epsilon} \, \overline{\nabla \cdot \vec{E}} \, = \, 4\pi \, \rho \, , \tag{10}$$

where $1 + \bar{\epsilon} \, = \, 1 + \epsilon(\omega \, = \, 0)$ is the dielectric constant at zero frequency, which, if large, suppresses the effects of homogeneous charge fluctuations. In any case, we could have started by postulating Eq. (8), and we will shortly consider yet another motivation for considering this equation of motion, in the context of a possible connection with the dS/CFT conjecture.

Note that the equation of motion (8) is consistent in the sense that it manifestly satisfies the Bianchi identities; this is true because for a given space-time $\bar{R}$ is simply a number, and therefore (since the metric is covariantly constant) the covariant divergence of the l.h.s. of the equation vanishes.

Clearly the definition of $\bar{R}$ is a formal one. There can be divergences in the integration both in the numerator and denominator, in infinite space-times or in the presence of short-distance curvature singularities. We can deal with the latter by excising regions of Planckian curvature from the space-time in the integrations. The former ambiguity can be dealt with in a broad class of examples. Clearly $\bar{R} = 0$ in any space-time with infinite volume but a finite integrated Ricci scalar. This includes any asymptotically flat space-time sprinkled with stars or black holes. $\bar{R}$ is also zero in a radiation, or matter-dominated, forever expanding Friedmann-Robertson-Walker (FRW) universe, because $\int dt R(t)$ converges (away from the big-bang singularity). In maximally symmetric dS or AdS spaces, R is constant, the numerator in (9) is R times the denominator, so it makes sense to define $\bar{R} = R$. Now consider a space-time that begins with a big-bang and is asymptotically dS with de Sitter curvature R_∞. Both the numerator and denominator are completely dominated by the (infinite) contributions from the asymptotic dS region in the future and therefore, in these space-times, $\bar{R}$ is reasonably defined to equal R_∞. These examples suffice for our immediate purposes, but it may turn out that more general prescriptions for defining $\bar{R}$ are needed in more interesting space-times.

Our modified equation of motion coincides with Einstein's equation for any system for which $\bar{R} = 0$. As we have just discussed this includes localised solutions such as a star, black hole, and matter- or radiation-dominated FRW cosmologies. Its main new physical consequence is that the enormity of $\bar{M}$ suppresses the value of $\bar{R}$, in spaces with non-vanishing $\bar{R}$, such as de Sitter space. This is accomplished without finely adjusting any of the parameters in the equation of motion. For simplicity, let us consider an energy-momentum tensor $T_{\mu\nu}$ that consists of a vacuum energy piece together with another contribution from radiation and non-relativistic matter

$$T_{\mu\nu} \,=\, g_{\mu\nu}\,\mathcal{E} \,+\, T^{\rm other}_{\mu\nu}\,. \tag{11}$$

Now, let us restrict our attention to space-times that begin with some generic big-bang singularity but are asymptotically de Sitter in the future. As we have argued, in such space-times, $\bar{R}$ is given by the asymptotic dS curvature, R_∞. We can self-consistently compute $\bar{R}$ by looking at the equation of motion in the deep future, where all sources of energy-momentum other than the vacuum energy have inflated away. Then we conclude that

$$\bar{R} \,=R_\infty = \,\frac{-4\,\mathcal{E}}{M^2_{\rm Pl}+\bar{M}^2}\,. \tag{12}$$

This is a tiny curvature if we make $\bar{M}^2$ large enough. For $\mathcal{E}$ near its smallest value compatible with naturalness, $\sim$ (TeV)^4, we need $\bar{M} \sim 10^{48}$ GeV in order to reproduce the observed acceleration of the Universe today. If $\mathcal{E}$ has the largest size $\sim M^4_{\rm Pl}$, then we need $\bar{M} \sim 10^{80}$ GeV, which is the mass of the Universe!

One may have thought that a natural value for $\bar{M}$ would be close to $M_{\rm Pl}$, but there is in fact no reason to believe this: $M_{\rm Pl}$ sets a short-distance physics scale, where gravity gets strongly coupled, while $\bar{M}$ clearly has to do with deep infrared, non-local physics. We already know that there is a large hierarchy between the weak scale and the Planck scale, so there may be a hierarchy between $M_{\rm Pl}$ and $\bar{M}$; it is of course tempting to speculate that in a fundamental theory these hierarchies are related. However, this large value of $\bar{M}$ does not need to be finely adjusted to any particular value in any sense.

We can not address the full issue of the radiative stability of these parameters since we do not have a full quantum theory of gravity; however, since we are modifying only the gravitational part of the equation of motion we can discuss radiative stability at least at the level of Standard Model radiative corrections. We assume $T_{\mu\nu}$ on the r.h.s. of the equation of motion to be derived in the usual way from the Standard Model quantum effective action $\Gamma_{\rm SM}[g]$, evaluated in a gravitational background $g_{\mu\nu}$:

$$T_{\mu\nu} \,=\, \frac{2}{\sqrt{g}}\,\frac{\delta\Gamma_{\rm SM}}{\delta g^{\mu\nu}}\,. \tag{13}$$

The various terms in $\Gamma_{\rm SM}$ are assumed to have their natural sizes. Since $\Gamma_{\rm SM}$ is obtained from quantum field theory loops and is completely local, there is no renormalization of $\bar{M}$ from Standard Model loops. In our considerations so far we have included the natural size of at least $\sim$ (TeV)^4 for the vacuum energy (as well as other sources of energy associated with, for instance, radiation, matter or the inflaton field). There are also other terms in this definition of $T_{\mu\nu}$, for example purely gravitational terms arising from SM loops with external graviton lines. However, all of these effects are absorbed into higher-dimension operators in $\Gamma_{\rm SM}$ suppressed by powers of $M_{\rm Pl}$. Given the enormity of $\bar{M}$ and the consequent miniscule size of $\bar{R}$, these operators have negligible effects suppressed by powers of $(\bar{R}/M_{\rm Pl}^2)$. The same stability arguments apply to the finite-L equation (2).

We can also see that our modification of gravity has no other effect in the $L \to \infty$ limit than to generate a miniscule *apparent* vacuum energy. We simply take the result for $\bar{R}$ and put it back into the equation; we have

$$M_{\rm Pl}^2\, G_{\mu\nu} \,=\, \frac{M_{\rm Pl}^2}{M_{\rm Pl}^2+\bar{M}^2}\,\mathcal{E}\, g_{\mu\nu} \,+\, T_{\mu\nu}^{\rm other}\,. \tag{14}$$

Therefore all other (good) properties of Einstein gravity are completely maintained. In particular, the standard slow-roll inflationary cosmology is untouched. We can have an inflaton with a potential satisfying the usual slow-roll condition, where the minimum of the inflaton potential is $\mathcal{E}$. All of inflation and the subsequent evolution of the Universe through reheating, nucleosynthesis, matter domination and structure formation would go through unchanged, and we would match it to a universe that would eventually be de Sitter with tiny curvature $\sim \mathcal{E}/\bar{M}^2$.

We can see this in a closely related way again by tracing and taking the space-time average of the l.h.s. and r.h.s. of Eq. (8), we arrive at

$$M_{\rm Pl}^2\, G_{\mu\nu} \,=\, T_{\mu\nu} \,-\, \frac{1}{4}\, g_{\mu\nu}\,\frac{\bar{M}^2}{M_{\rm Pl}^2+\bar{M}^2}\bar{T}\,, \tag{15}$$

where $\bar{T} \equiv \int d^4x\sqrt{g}T/\int d^4x\sqrt{g}$. Here we are subtracting the space-time average of T on the r.h.s. of the Einstein equation, which has the effect of subtracting out the vacuum energy in an asymptotically dS universe. Note that for $\bar{M}^2 \gg M_{\rm Pl}^2$, the coefficient of the second term is extremely close to 1. Had we simply written down this equation of motion to begin with, with $\bar{M}^2/(\bar{M}^2+M_{\rm Pl}^2)$ replaced by a parameter x, then in order to address the CCP we would have to adjust x to be very close to 1 with enormous accuracy. (The case with $x = 1$ is reminiscent of the proposal of [4]). However, what we have seen is that this can be a consequence of Eq. (8), where there are no fine adjustments at all but simply large hierarchies between $M_{\rm Pl}$, $\bar{M}$ and $\mathcal{E}$. Since the form of the equation of motion given in (8) is free from fine adjustments, then this is what we should try and match from a more fundamental theory.

Note that it is impossible to find solutions which are asymptotically *flat* in the future; any such solution would have $\bar{R} = 0$, and be in contradiction with the r.h.s. of Eq. (14) for non-zero $\mathcal{E}$. Thus, asymptotically flat spaces can only arise in theories where $\mathcal{E} = 0$, such as in large classes of supersymmetric models: in those cases of course no asymptotically de Sitter solutions would exist.

It is interesting to compare our approach with "unimodular gravity". In order to do so, we rewrite Eq. (8) as a system of the following two equations:

$$M_{\rm Pl}^2 \left(R_{\mu\nu} - \frac{1}{4}\, g_{\mu\nu}\, R \right) = T_{\mu\nu} - \frac{1}{4}\, g_{\mu\nu}\, T \,, \tag{16}$$

$$M_{\rm Pl}^2\, R + T = -\, \bar{M}^2\, \bar{R} \,, \tag{17}$$

where $T \equiv T^\alpha_\alpha$ denotes the trace of the energy-momentum tensor.

The first equation is that of unimodular gravity (see, e.g., [1]). In addition we get the second equation, which plays a vital role. To see this let us start with $T_{\mu\nu} = \mathcal{E}\, g_{\mu\nu}$. For this stress tensor the r.h.s. as well as the l.h.s. of Eq. (16) is zero identically. Therefore, Eq. (16) alone does not determine the curvature. To find the curvature we turn to Eq. (17). The latter gives Eq. (12). Since $\bar{R}$ is a space-time constant the Bianchi identities are trivially satisfied for Eq. (8), and for the system (16)–(17) too. We act on both sides of Eq. (16) by the covariant derivative ∇^μ. Since the energy-momentum tensor of matter and the Einstein tensor are covariantly conserved, $\nabla^\mu T_{\mu\nu} = \nabla^\mu G_{\mu\nu} = 0$, we obtain

$$\partial_\mu \left(M_{\rm Pl}^2\, R + T \right) = 0 \,. \tag{18}$$

This equation implies that $M_{\rm Pl}^2\, R + T$ can be an arbitrary space-time constant as in unimodular gravity [1]. However, the vital ingredient of the present approach is the second equation, (17), which uniquely determines the value of curvature to be in agreement with (12) in asymptotically de Sitter space-times.

It is likely that there are other solutions to our equations of motion, which are not asymptotically dS; and which for instance correspond to finite-volume "bang-crunch" type cosmologies. However, it is clear that these solutions are not in any sense continuously connected with the desirable ones that *are* asymptotically de Sitter. Further, as we will discuss below, there may be more fundamental reasons for restricting ourselves to asymptotically de Sitter universes.

3.2 Acausality Instead of Fine-Tuning

Perhaps the most disturbing feature of our modification to Einstein's equations is that it is manifestly acausal. Therefore, it is crucial to understand whether this may affect observations. We will argue below that the acausality has no significant effect on any observable source in the Universe, while it is very important for solving the CCP.

A simple argument suggests that if some sort of non-local modification of gravity is responsible for resolving the CCP in a realistic setting, it should have acausality as a fundamental feature. Imagine a *causal* modification of gravity in the infrared, at some scale $\sim L$, such that while energy-momentum localised in space or time over scales smaller than L gravitate normally, energy-momentum spread out over scales much larger than L hardly gravitate. For our Universe, we would have to require that L be at least of the order the size of our Universe today, $\sim 10^{28}$ cm. Now suppose that the Universe begins in a big bang at some time moment $t = 0$; and the energy-momentum tensor has a vacuum energy component that we are trying to render harmless by our IR modification. But how can *causal* physics know whether the cosmological constant is truly *constant* and not some temporary blip, as in inflation? If it *were* a blip, which disappeared after a time much smaller than L, it would have to gravitate normally, and therefore inflate. It would then take causal physics a time of order L, which is at least ten-billion years, to recognise that the cosmological constant was truly constant; only then would the large rate of inflation cease. But for our Universe, there are observational reasons to believe that this can not happen. The successes of standard cosmology starting from big-bang nucleosynthesis at 1 second, to the present, indicate that the Universe must have been "normal" since it was one second old. There may be loopholes around this argument, however the issue of the time scale needed to cancel the CC must clearly be addressed in any scenario with causal modifications of gravity.

By contrast, our non-local modification is maximally acausal in the interesting case of an asymptotically de Sitter universe, because it is dominated by the deep *future* behaviour of the geometry. However, this acausality does not lead to any peculiar behaviour (other than the suppression of the effective CC!), precisely because the deep future behaviour in a de Sitter space is so universal. In fact, we have seen that after self-consistently solving Eq. (8) for $\bar{R}$ and inserting the solution back into the equation of motion, we have a completely *local* equation of motion (14), where, however, the vacuum energy part of the stress tensor seems to be unnaturally small! Therefore, at least for asymptotically de Sitter spaces, there are two descriptions of the physics: one which is free of any fine-tuning but highly acausal, and another which is *local* and *causal*, but which appears to have a highly unnatural value for the vacuum energy.

The acausality also takes care of one of the usual conundrums associated with attempting to make vacuum energy "not gravitate". Locally in time, we just have some energy momentum tensor $T_{\mu\nu}$ composed of, say, contributions from matter and radiation as well as $\mathcal{E} g_{\mu\nu}$. How can we disentangle the "vacuum energy" part of $T_{\mu\nu}$ and tell it not to gravitate? Local physics can not do this without modifying the response of gravity to either matter or radiation as well; however, our acausal modification knows how to do this. The vacuum energy part of $T_{\mu\nu}$ is precisely the part that does not dilute away in an asymptotically de Sitter universe deep in the future.

Finally, the acausality also resolves another minor puzzle: as we have seen, the $L \to \infty$ of Eq. (2) does not lead to Einstein's theory. This seems to violate naive decoupling intuition. The physical reason that this does not violate decoupling is that L is not the true infrared scale of the theory. There is an even longer time scale, the (infinite) total age and size of the asymptotically de Sitter universe.

3.3 Possible Connection with dS/CFT

There is a quite different way of motivating our equation of motion (8), which resonates with some interesting qualitative ideas springing from the dS/CFT conjecture [9].

Our present formulation of the equations of motion (8), involving the space-time average of R, gives rise to perfectly satisfactory physics for asymptotically de Sitter spaces, though, as we mentioned, likely there are other solutions that are not asymptotically dS. Actually, since our equation is non-local in time in any case, we could declare that we are *only* interested in universes that are asymptotically dS in the future. This may seem like a perverse thing to do at first, but it is quite sensible when viewed in the context of the dS/CFT conjecture. In this correspondence, there is a dual description of physics in asymptotically de Sitter spaces, in terms of some sort of Euclidean conformal field theory in one lower dimension without gravity. A concrete example of such a theory is still lacking, but if one existed, we would be tempted to say that it is the fundamental definition of quantum gravity in asymptotically de Sitter space-times, much as $\mathcal{N} = 4$ SYM is taken to be the fundamental description of backgrounds in string theory that are asymptotic to $AdS_5 \times S_5$.

In this correspondence, time is the holographically generated dimension and is associated with the RG scale in the dual theory; with the *ultraviolet* of the dual field theory identified with the *deep future* in the space-time description. the RG flow from the UV to IR is associated with inverse time evolution in the bulk. For instance, an RG flow in the field theory that starts near a UV fixed point, and skims by an IR fixed point before developing a mass gap and becoming free in the deep IR, plausibly has a space-time description in terms of a universe that begins in a big bang, goes through an inflationary epoch (near the IR fixed point) and ends in an accelerating dS phase (UV fixed point). It is fascinating that in this picture, the fundamental degrees of freedom, corresponding to the deep UV of the dual theory, are associated not with the big bang but with the *deep future* dS phase of the universe.

If we view the history of our own Universe in terms of RG flow in some dual CFT, there is then simply no choice about the deep future behaviour of the Universe; the fundamental (UV) theory is dual to a space-time that is asymptotically de Sitter in the future. However, the CCP still exists: Why is the asymptotic de Sitter radius so much larger than the naive expectation from the Standard Model vacuum energy?

As we have seen, we can rephrase the question. The conundrum only results from assuming that the effective space-time description of this RG flow is given by the (local) Einstein equations. But this has not been derived at the current level of understanding of quantum gravity in de Sitter space in general, or of dS/CFT in particular. We can therefore entertain the possibility that the effective classical space-time description of the RG flow differs from Einstein's equations, in such a way that the largeness of the asymptotic de Sitter radius does not appear finely tuned. An obvious possibility is

$$M_{\rm Pl}^2\, G_{\mu\nu} \,-\, \frac{1}{4}\, \bar{M}^2\, g_{\mu\nu}\, R_\infty \;=\; T_{\mu\nu}\,, \tag{19}$$

where R_∞ is now simply defined as the asymptotic dS curvature, rather than in terms of any space-time average. In fact, R_∞ is a natural quantity in dS/CFT, because the UV value $c_{\rm UV}$ of the CFT central charge—essentially the number of degrees of freedom in the CFT—is determined by the curvature in Planck units as

$$c_{\rm UV} \;=\; \frac{M_{\rm Pl}^2}{|R_\infty|}\,. \tag{20}$$

At any rate, all the consequences of Eq. (19) are identical to what we have already seen, and again, we can deduce an equivalent description which is local but where the CC appears absurdly finely tuned. However, the fundamental starting point and interpretation here are rather different.

Since the dual theory seems to have only one relevant large dimensionless number, the central charge $c_{\rm UV}$, it is tempting to imagine that the parameters $\bar{M}$ and the vacuum energy $\mathcal{E}$ are both determined in terms of $c_{\rm UV}$, in such a way that the produced $\bar{M}$ is huge with respect to the Planck scale, while also producing a small Standard Model vacuum energy (which in a supersymmetric theory would be related to the electroweak scale in the usual way). Such a correlation offers a possible simultaneous solution to the CCP as well as the hierarchy problem, and may be related to the proposal of Banks [10]. It can also explain the oft-noted striking "co-incidence" that the weak, Planck and Hubble scales appear to be related as $H^2 \sim m_{\rm EW}^8/M_{\rm Pl}^6 \sim \mathcal{E}^2/M_{\rm Pl}^6$. This can arise if we take $\bar{M}$ and $\mathcal{E}$ to scale with $c_{\rm UV}$ as

$$\bar{M} \sim c_{\rm UV}^{1/2}\, M_{\rm Pl},\;\; \mathcal{E} \sim c_{\rm UV}^{-1/2}\, M_{\rm Pl}^4\,, \tag{21}$$

which yields

$$\frac{H^2}{M_{\rm Pl}^2} \;=\; \frac{R_\infty}{M_{\rm Pl}^2} \;=\; \frac{1}{c_{\rm UV}} \;=\; \frac{\mathcal{E}^2}{M_{\rm Pl}^8}\,. \tag{22}$$

4 Finite L Theories

We have seen that addressing the CCP in our approach can already be done in the $L \to \infty$ limit, where the only remnant of our non-local modification, at least in asymptotically de Sitter space-times, is the suppression of the cosmological constant. But there are clearly potentially new phenomena associated with finite L theories, and in any case finite L acts as a regulator that can shed some more light on the general mechanism for suppressing the CC.

4.1 Toy Scalar Example

Let us begin by considering a toy scalar example of a finite L theory, where the complications of non-linearity and tensor structure of a full gravitational theory are removed, but where much of the physics associated with our non-local modifications remain. This example is essentially the linearised gravity approximation without the indices. To begin with, consider a scalar field ϕ coupled to a source T. The equation of motion is

$$\nabla^2\phi + T = 0 . \tag{23}$$

We can think of

$$R_{\rm toy} \equiv \nabla^2\phi , \tag{24}$$

as our toy analog of the gravitational curvature.

In this toy world, the field ϕ couples to matter and mediates long-range scalar gravity, which has been successfully "measured" to distance scales $\sim H_0^{-1}$; also the "gravitation" of spatially homogeneous sources has been measured for times as long as $\sim H_0^{-1}$. The toy analogue of the cosmological constant problem is that the source T is also expected to contain a large space-time homogeneous component $\mathcal{E}_{\rm toy}$, but the measured value of $R_{\rm toy}$ is far smaller than $-\mathcal{E}_{\rm toy}$. This can be remedied by modifying the equation of motion as

$$\Big(1 + \mathcal{F}(L^2\nabla^2)\Big)\, R_{\rm toy} + T = 0 . \tag{25}$$

We would like to ensure that this modification removes the toy CC problem without adversely affecting anything else. This will require constraints on the function $\mathcal{F}$ in addition to the basic requirements that $\mathcal{F}(\alpha) \to 0$ for $\alpha \gg 1, \mathcal{F}(0) \gg 1$. Passing to momentum space, we find that

$$\Big(1 + \mathcal{F}(L^2p^2)\Big)\, \tilde{R}_{\rm toy}(p) + \tilde{T}(p) = 0 . \tag{26}$$

At generic momenta we can simply divide by $(1 + \mathcal{F}(L^2p^2))$ and conclude that

$$\tilde{R}_{\rm toy} = -\frac{\tilde{T}(p)}{1 + \mathcal{F}(L^2p^2)} . \tag{27}$$

For a constant source, $\tilde{T}(p) = \mathcal{E}\delta(p)$, and in the limit where $\mathcal{F}(0) \to \infty$, we can make $R_{\rm toy}$ vanish. Phrased diagrammatically, we have modified the ϕ propagator to vanish at zero external momentum, and therefore the T tadpole does not force any space-time variation for ϕ, and so ϕ can be put at any constant value ϕ_0. Furthermore, as long as $(1 + \mathcal{F}(L^2p^2))$ rapidly approaches 1 for real $|p^2| > H_0^2$, we are guaranteed that the successes of our toy gravity are maintained.

In general, the function $(1 + \mathcal{F}(z))$ may have zeros in the complex z at some points z_i; if so, there will be new solutions to the homogeneous equations of motion with effective frequency $\omega_i^2 = \frac{1}{L^2} z_i$. The existence of such "transients" is disastrous because it implies that there are solutions

$$R_{\rm toy} \;\sim\; R_0\, e^{-i\omega_i t}\,, \tag{28}$$

where R_0 is *any* initial amplitude. Thus, while it may be impossible to find solutions for $R_{\rm toy}$ that are large and *exactly* stationary, there are infinitely many solutions where $R_{\rm toy}$ has arbitrarily large amplitude, varying very slowly over time scales of order $\sim L$. In a realistic gravitational setting, these would correspond to universes that are, for example, inflating, but with the rate of inflation very slowly changing [1]. To see this, consider a space-time where $R \sim \bar{R} + H^2 e^{-i\omega t}$; Here, the $\bar{R}$ is a particular solution of Eq. (2) with a constant energy density on its r.h.s., while the parameter H can take any value. Since the $\bar{R}$ is tiny (because $\bar{M}$ is large), and if ω is very small compared to H, then in computing $(1+\mathcal{F}(L^2\nabla^2))R$, to a good approximation we can use the metric with constant H in ∇^2. Thus, $\nabla^2 \simeq \partial_t^2 + 3H\partial_t$ and we obtain

$$(1 + \mathcal{F}(L^2\nabla^2))\, H^2 e^{-i\omega t} \;\sim\; (1 + \mathcal{F}(-L^2\omega^2 + 3iHL^2\omega))\, H^2\, e^{-i\omega t}\,. \tag{29}$$

Therefore, if $(1+\mathcal{F}(z))$ has roots in the complex plane, there will be solutions which are inflating with a rate H which is very slowly changing on a time-scale of HL^2 or L, whichever is larger. We would then have to explain why the Universe is not in one of these solutions, converting the fine-tuning problem for the CC into an essentially equally finely-tuned question about initial conditions. It is therefore desirable to eliminate such transients by insisting that $(1 + \mathcal{F}(z))$ have no zeros in the complex plane (except at infinity).

An explicit example of a function that has these properties is

$$1 \,+\, \mathcal{F}(L^2p^2) \;=\; \frac{1}{1+\epsilon \,-\, e^{-(L^2\,p^2)^2}}\,, \tag{30}$$

which quickly asymptotes to 1 as $|p^2|L^2$ gets large, satisfies $\mathcal{F}(0) \to \epsilon^{-1} \simeq \bar{M}^2/M_{\rm Pl}^2$, and has no zeros at any finite point in the complex plane. Another example that satisfies all the above requirements is

$$1 \,+\, \mathcal{F}(L^2p^2) \;=\; \exp\left(N e^{-(L^2\,p^2)^2}\right)\,, \tag{31}$$

[1] We thank Maxim Perelstein for discussions on this point.

where $N \simeq \ln(\bar{M}^2/M_{\rm Pl}^2)$ is some large number.

Let us restrict ourselves to sources that are spatially homogeneous. Then, we can solve for the toy curvature as a function of time

$$R_{\rm toy}(t) \,=\, -T_{\rm eff}(t)\,, \tag{32}$$

where

$$T_{\rm eff}(t) \,=\, T(t) \,-\, \int dt' K(t-t')\, T(t')\,, \tag{33}$$

with the kernel K given by

$$K(t) \,=\, \int d\omega\, f(L^2\omega^2)\, e^{-i\omega t}\,. \tag{34}$$

Here the function f is defined as $(1+\mathcal{F})^{-1} \equiv (1-f)$. Note that since we have assumed that $1+\mathcal{F}$ has no zeros, $(1-f)$ has no poles on the real axis. Furthermore, $f(\alpha)$ quickly asymptotes to zero for large α, while it approaches 1 near $\alpha = 0$. Therefore, the integrand in the definition of $K(t)$ is regular. Since f is a function of ω^2, K is manifestly time-symmetric, $K(t) = K(-t)$ and is therefore acausal. Therefore we conclude that if we wish to avoid the existence of the undesirable transient solutions, we are *forced* to have acausality. As we have argued in the previous section, however, such acausality is actually welcome for a realistic solution of the CCP.

Note as well, that as $L \to \infty$, $\int dt' K(t-t')T(t')$ is just $f(0)$ times the time average of the source T. For $f(0) = 1$ and a constant T, $T_{\rm eff}$ vanishes. Now consider a source which is a step function, $T(t) = T_1$ for $t < 0$, $T(t) = T_2$ for $t > 0$. This is the toy analog of an inflationary phase transition from one vacuum energy to another. Here, for $L \to \infty$ and $f(0) = 1$, we have $T_{\rm eff} = T(t) - (T_1 + T_2)/2$. The effective source is not cancelled for either $t > 0$ or $t < 0$. This is because the source is uniform in both the deep past and the deep future, and the space-time average is thus equally weighted by past and future values, and does not cancel either of them. This behavior would be a disaster for our real gravitational case of interest, but fortunately, our toy example fails as a good analogy here for the following two reasons. First, any realistic cosmology has an origin of time in the big-bang, and therefore the inflationary potential does not dominate the energy for infinite times into the past, while in an asymptotic de Sitter universe the vacuum energy persists infinitely into the future. The average is then dominated by the deep future value of the vacuum energy, which is what is effectively cancelled (or suppressed for large but not infinite $\bar{M}$). Second, the curved space d'Alambertian, ∇^2, introduces a new scale in the complete theory—the Hubble parameter of the corresponding time-dependent background H, or square root of the scalar curvature for static backgrounds. This scale enters the argument of the $\mathcal{F}$ and plays a vital role in calculations. Due to the presence of that scale there is no delay by L in the response of the $\mathcal{F}$ to any sudden change in a source, such as, e.g., the change during phase transitions.

We now want to consider finite L modifications for real gravity, by returning to Eq. (2).

4.2 Bianchi Identities

Let us act on both sides of Eq. (2) by ∇^μ. Since $\nabla^\mu G_{\mu\nu} = \nabla^\mu T_{\mu\nu} = 0$ we get

$$\nabla^\mu \left(\mathcal{F}(\nabla^2)\, G_{\mu\nu}\right) = 0 \,. \tag{35}$$

The covariant derivative does not commute with ∇^2 for general backgrounds; therefore, (35) is an *additional* constraint on $G_{\mu\nu}$. Hence, the Bianchi identities are not just kinematically satisfied as in the Einstein gravity. Instead, all possible consistent solutions of Eq. (2) should satisfy the new constraint (35). It is of course possible that there is a more clever non-local modification that kinematically satisfies the Bianchi identities, but let us press on. Note that the same conclusion can be derived by inverting the $1+\mathcal{F}$ operator in Eq. (2) and putting it to the r.h.s. Then, the Bianchi identities lead to the following constraint

$$\nabla^\mu \left(\frac{1}{1\,+\,\mathcal{F}(L^2\nabla^2)}\, T_{\mu\nu}\right) = 0 \,. \tag{36}$$

For a given source $T_{\mu\nu}$ this equation should be interpreted as a new constraint on the corresponding metric that enters the covariant derivatives in (36), and not as a constraint on the source itself.

Note that the tensor $\tau_{\mu\nu} \equiv (1\,+\,\mathcal{F}(L^2\nabla^2))^{-1} T_{\mu\nu}$ is covariantly conserved according to Eq. (36). Order by order in perturbations, we can try to restore an action that would give rise to $\tau_{\mu\nu}$ as its stress-tensor. There are two important things about that action. First, it contains an infinite number of local terms that cannot be truncated at any finite order; therefore, the would-be action is fundamentally non-local. Second, we notice that this action contains additional vertices of interactions of gravitational perturbations with matter fields in $T_{\mu\nu}$. These vertices may play a crucial role in certain processes discussed below.

Let us start with perturbations on a flat background for which $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. We would like to check what are the new constraints imposed by (35) on these perturbations. In the linearised approximation Eq. (35) is satisfied if $\partial^\mu h_{\mu\nu} = \partial_\nu h^\alpha_\alpha/2$. The latter expression is nothing but the *harmonic* gauge-fixing condition in the Einstein gravity. Hence, in the linearised approximation the constraint (35) does not give rise to any new restrictions on perturbations.

As a next step, we write down the expression for the response of gravitational field to a source with the stress-tensor $T_{\mu\nu}$. From (2) we derive

$$h_{\mu\nu} = \frac{8\,\pi\, G_N}{[1+\mathcal{F}(\nabla^2)]\,\nabla^2} \left(T_{\mu\nu} - \frac{1}{2}\,\eta_{\mu\nu}\, T^\alpha_\alpha\right) . \tag{37}$$

The novelty is the appearance of $(1+\mathcal{F})^{-1}$ on the r.h.s. This suppresses the gravitational response to states with $p^2 = \omega^2 - \vec{p}^2 \sim 0$. The role of such states in ordinary astrophysical objects, such as stars and planets, is negligible. Therefore, we do not expect any substantial modification of gravitational fields of classical astrophysical objects. Note that a massless particle that is exactly on shell will generate a suppressed gravitational field. However, any massless particle that is emitted and absorbed over scales smaller than L is always off-shell by an amount greater than $1/L$, and will therefore gravitate normally.

Note also that, in this model, action and reaction are not necessarily equal. To see this let us calculate the interaction between sources $T_{\mu\nu}$ and $T^{(1)}_{\mu\nu}$. In the lowest approximation in G_N the source $T_{\mu\nu}$ sets the gravitational field given in Eq. (37). The interaction with $T^{(1)}_{\mu\nu}$ is proportional to $h^{\mu\nu}T^{(1)}_{\mu\nu}$. However, the latter expression is not symmetric w.r.t. the interchange of the sources. Hence, the action of $T_{\mu\nu}$ does not equal the reaction of $T^{(1)}_{\mu\nu}$ and *vice versa*. As in electrodynamics, this indicates that there should be some radiation that accounts for the mismatch between the action and reaction. Let us call it the *L-radiation*. Perhaps this is gravitational radiation due to the new vertices that appear in the covariantly conserved stress tensor $\tau_{\mu\nu} \equiv (1 + \mathcal{F}(L^2\nabla^2))^{-1}T_{\mu\nu}$, which we discussed above. Alternatively, this may be interpreted as a new form of radiation of states that should be "integrated in" in order to make our action local. The vacuum energy $\mathcal{E}$ in our case does not act on matter in the Universe since the former sets no gravitational field. However, the matter does act on the vacuum. As a result, there should be the compensating L-radiation from the vacuum, especially in the regions in the Universe that have high matter density.

Finally all known non-linear solutions should satisfy (35). For the Schwarzschild solution, $G_{\mu\nu} = 0$ everywhere outside the source. Therefore it trivially satisfies (35). Inside the source, however, the standard solution will be modified by a quantity that vanishes in the limit $L \to \infty$. Equation (2) has also an exact dS-Schwarzschild solution outside of the source [2]. The gravitational radius in that solution is proportional to the conventional Newton coupling G_N, while the curvature is proportional to an effective coupling $G_N/(1 + \mathcal{F}(0))$.

4.3 Standard Astrophysics and Cosmology

Since our proposal modifies gravity, it is important to ensure that it does not alter standard astrophysics and cosmology. As long as the scale L is larger than the present size of the observable Universe $L \gtrsim 10^{28}$ cm, it is clear that astrophysics will not be affected because the relevant length scales are shorter than L and consequently the filter function vanishes. Similarly, early cosmology will not change because the relevant length scales—horizon size, particles' Compton wavelengths, and inverse temperature—are all shorter than L.

Mathematically we can see this by decomposing the stress tensor into its vacuum energy piece and the rest (matter plus radiation):

$$T_{\mu\nu} = \mathcal{E}\, g_{\mu\nu} + T'_{\mu\nu}\,, \tag{38}$$

[2] We thank Nemanja Kaloper for pointing this out to us.

where $T'_{\mu\nu}$ denotes the stress tensor for everything but the vacuum energy. The solution of Eq. (2) with (38) on the r.h.s. takes the form

$$G_{\mu\nu} \,=\, \Lambda\, g_{\mu\nu} \,+\, G'_{\mu\nu}\,, \tag{39}$$

where Λ is defined by

$$\Lambda \,\equiv\, \frac{\mathcal{E}}{M_{\rm Pl}^2\,[1+\mathcal{F}(0)]} \ll \frac{\mathcal{E}}{M_{\rm Pl}^2}\,, \tag{40}$$

and $G'_{\mu\nu}$ satisfies the equation

$$M_{\rm Pl}^2\, G'_{\mu\nu} \,\simeq\, T'_{\mu\nu}\,. \tag{41}$$

To obtain the latter expression we used $\mathcal{F}(L^2\,\nabla^2)G'_{\mu\nu} \simeq \mathcal{F}(\text{argument} \gg 1)G'_{\mu\nu} \simeq 0$. Equation (41) is the conventional Einstein equation on the background with the vanishingly small cosmological constant (40).

So, the pure vacuum energy gravitates with $G_N^{\rm eff} \equiv G_N/(1+\mathcal{F}(0)) \ll G_N$, while the matter and radiation in the Universe gravitate with the conventional Newton constant G_N. As a result, early cosmology remains unaffected, while eventually the small gravity of vacuum takes over and dominates the dynamics of the Universe. Since we do not have a theory predicting the value of the filter function, we cannot explain the observational fact why this is happening in our epoch.

4.4 Inflation and Exit

It is natural to worry that theories addressing the CCP might have unwanted consequences for inflation [11, 12, 13, 14, 15]. First, inflation is driven by vacuum energy which may not gravitate in such theories. Second, inflation ends when the Universe transitions to the "true vacuum" which is normally assumed to have zero energy — an assumption that is now replaced by a dynamical principle. In this section we argue that our proposed modifications of general relativity maintain the successes of some of the existing inflationary scenaria — notably "new" and chaotic inflation— while suggesting new possibilities for simple and perhaps more natural inflationary theories.

It is clear that our modified equations differ from Einstein's only for systems that are simultaneously slow and big compared to L. Conversely, systems that are either fast or small compared to L behave according to the familiar laws of general relativity. An example of such a system is the Universe inflating according (now old) "new inflation" paradigm [12, 13]. There, the inflationary phase is driven by a scalar field rolling with a characteristic time scale much shorter than L, and is therefore unaffected by our modifications. The same is true for chaotic inflation [14] in which an overdamped scalar field rolls, again at a rate fast compared to L^{-1}. The subsequent phases of exiting inflation and reheating also occur on time-scales shorter than L. So, the only effect of our modification, in either new or chaotic inflation, is to "filter out" the (arbitrary) constant part of the potential, as desired.

To find a novel inflationary scenario — one where our modifications make a difference— we first have to look for a system that is slow compared to L. A simple example is a classical scalar field at rest at some local minimum of its potential (not necessarily the true minimum). This suggests the following possibility: Consider a false vacuum bubble (or island) of size d and positive energy density V, created at time $t = 0$ by tunnelling from the true vacuum. The Fourier components of such a potential at time $t = 0$ have characteristic wavelengths of order d, the initial size of the system. Suppose that $M_{\rm Pl}/\sqrt{V} \ll d \ll L$. Then, the region of space within the island will start to inflate with the conventional rate and the characteristic wavelengths of the system will get red-shifted, and—one by one, longer ones first—will get stretched beyond L. As a result, these wavelengths become decoupled from gravity or "degravitated". This gradual peeling-off of the gravitating Fourier components ends when the whole potential has been degravitated causing inflation to terminate. The above mechanism of "self-termination" for exiting inflation is inherent in our framework. It naturally leads to the final state of very weakly gravitating vacuum energy.

To decorate these ideas with equations, let us study the behavior of the scalar curvature during inflation. The trace of the modified Einstein equation takes the form

$$-M_{\rm Pl}^2 \left(1 \,+\, F(L^2\,\nabla^2)\right)\, R(t,\vec{x}) \;=\; T(t,\vec{x})\,. \tag{42}$$

Concentrate now on a small region of size $|\Delta x| \ll d$ inside the island. Since the boundary effects are negligible in that region, we expect that the curvature there is approximately constant (to an accuracy of $\mathcal{O}(|\Delta x|/d)$). Therefore, the metric in that region can be approximated by the standard ansatz, $ds^2 = dt^2 - a^2(t)d\vec{x}^2$. Then, from (42) we obtain

$$R \simeq -\frac{4V}{M_{\rm Pl}^2}\left\{1 \,+\, \mathcal{F}\left(\frac{L^2}{d^2a^2}\right)\right\}^{-1}\,, \tag{43}$$

where we used the fact that the characteristic (covariant) momentum square in the bubble of the initial size d scales as $|k_*^2| \sim g^{ii}/d^2 = 1/d^2a^2(t)$.

As long as the argument of $\mathcal{F}$ is larger than unity, we are back to general relativity. However, if the scale factor a grows with time, there comes a moment $t = t_0$ after which $L \lesssim da(t_0)$. Thus, for $t > t_0$ the argument of the $\mathcal{F}$ function drops below unity and the denominator of Eq. (43) becomes enormous. As a result, for $t \gtrsim t_0$ inflation continues with a very small rate that is suppressed by $\mathcal{F}(0)$. Since for an inflationary potential $a \sim \exp(Ht)$, the period of rapid inflation ends when

$$t \,\gtrsim\, t_0 \,\sim\, \frac{1}{H}\ln(k_*\,L)\,, \tag{44}$$

where k_* denotes the characteristic physical momentum in the initial state of the system. To summarise: the system undergoes rapid inflation for the period of time $0 < t < t_0$, after which it settles to the reduced inflation rate:

$$H_{\rm reduced} = \left(\frac{V}{M_{\rm Pl}^2 \left[1 + \mathcal{F}(0) \right]} \right)^{1/2} . \tag{45}$$

The present-day acceleration of the Universe determines $\mathcal{F}(0)$.

So far we have shown how degravitating can lead to a new exit from inflation that is inherent in our framework and does not make use of the detailed shape of the potential. A realistic inflationary model must also provide a mechanism for reheating, to produce matter in the Universe. One possibility that is generic is reheating due to particle creation [16]. If the change in curvature due to our mechanism is abrupt this results in the production of pairs of particles. The pairs can reheat the Universe provided that they are not redshifted by the expansion. From Eq. (44) we deduce that the time scale for changing the curvature is

$$\Delta t = \frac{1}{H} \frac{|\Delta k|}{k_*} , \tag{46}$$

where $|\Delta k| \sim 1/d$ is the band of wave-vectors associated with the initial island. To avoid total redshift of the created pairs, Δt has to be shorter then the doubling time of the universe, $\Delta t < H^{-1}$. Combining this with (46) we find the mild condition $|\Delta k| < k_*$.

Therefore, in our framework, if this condition is satisfied, the island (or bubble) of constant potential provides us with a scenario that has inflation, exit, reheating as well as eternal acceleration at the reduced rate of Eq. (45). Of course, the arguments presented above are a sketch of real computations that have to be done properly to see if these ideas are completely viable.

Note that in the conventional approach this scenario would have been impossible since there is no homogeneous classical process in general relativity that would end inflation and lead to reheating in such a uniform island. One conceivable mechanism is inhomogeneous and involves quantum-mechanical bubble nucleation, as was proposed by Guth in his original inflationary scenario [11]. Unfortunately, within the framework of conventional general relativity this scenario is excluded [17]. However, we can use the degravitation mechanism described above to end Guth's inflation without invoking the bubble nucleation process that causes problems in Guth's original proposal. If this is the way things are, we still live inside an island of false vacuum and all matter in the Universe originated in the Hawking radiation that got converted into matter by the sudden self-termination of inflation.

5 Discussion and Outlook

It is likely that the ultimate solution to the cosmological constant problem will require a major shift from some currently cherished physical principles. Given that the problem is associated with far-infrared scales, locality is a natural target to be sacrificed. But how can this concretely address the problem, and why does the world appear at least approximately local? In this paper we have attempted to address these questions at a long-wavelength, classical level, by presenting simple, non-local modifications of Einstein's equation that dramatically weaken the gravitational effect of vacuum energy, while preserving the usual successes of general relativity, so that a natural vacuum energy of size $\sim$ (TeV)^4 or even (10^{19} GeV)^4 does not lead to unacceptably large curvature. The modification can be qualitatively thought of as making the Planck scale enormous for Fourier modes with wavelength larger than some scale L. This weakens the effect of sources of energy-momentum uniform in space and time, like the vacuum energy, while leaving the gravitational effect of other sources unaffected.

In the $L \to \infty$ limit, the Einstein equation is modified in a universal way, by the addition of a term $\bar{M}^2 g_{\mu\nu}\bar{R}$, where $\bar{R}$ is the space-time average of the Ricci scalar. This term is not only non-local but also acausal. Nevertheless, in a broad class of space-times which become asymptotically de Sitter in the future, the entire effect of this modification is absorbed into making the asymptotic de Sitter curvature tiny. The acausality is a crucial ingredient, because it is the infinite asymptotic de Sitter future that dominates $\bar{R}$ and leads to a suppression of the asymptotic dS curvature. However, while the fundamental equations have a large non-local piece and a natural size for the Standard Model vacuum energy, in asymptotically de Sitter space-times there is an equivalent description of the physics which is local but where the effective vacuum energy appears un-naturally small. This example shows explicitly how a non-local effect might address the CCP, while having the rest of physics look local, reproducing all the usual successes of general relativity. Of course, in order for the curvature to be sufficiently small, the scale $\bar{M}$ must be extremely large, $\bar{M} \sim 10^{48}$ GeV for (TeV)^4 vacuum energy density, or the mass of the Universe $\bar{M} \sim 10^{80}$ GeV, for Planckian vacuum energy density. However, this large value is stable under Standard Model radiative corrections, which do not generate non-local operators with enormous coefficients. In this set-up, it is possible to imagine a common solution to both the CCP and the hierarchy problem, if the same physics simultaneously generates the enormous $\bar{M}$ and the tiny supersymmetry breaking with respect to the Planck scale.

For finite L, a host of exciting new physical phenomena arise, including new possibilities for inflation and the exit from inflation as inflating bubbles stretch to sizes greater than L and stop gravitating. This opens up new directions to explore both in the context of standard inflationary scenarios such as new inflation, chaotic or hybrid inflation, and motivates a re-examination of Guth's old inflation scenario as well. Further exploration of these ideas, even at this phenomenological level, could lead to observable consequences that can be looked for in the CMBR.

There are clearly a large number of avenues to explore. Theoretically, the obvious outstanding issue is to find a fundamental theory that reproduces our equation of motion in a classical, long-wavelength approximation. Such a theory must somehow incorporate the ingredients of non-locality and acausality in a consistent way, and also lead to an understanding of the large size of $\bar{M}$ with respect to the Planck scale. But even at the level of our phenomenological equations of motion, many issues need to be settled. For instance, what do the solutions of Eq. (8) look like which are not asymptotically de Sitter? How do we sensibly define $\bar{R}$ in universes with a complicated global structure? This question is particularly relevant in inflationary scenaria, once quantum effects for the matter fields are taken into account, which allow quantum fluctuations of the inflaton back up its potential hill leading to "eternal inflation". Another question along the same lines is: what happens to false vacuum inflation once tunnelling is taken into account? There are similar and even more interesting questions for the finite-L scenario, together with possibly observable phenomenological consequences to be explored.

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